History and Epistemology in Mathematics Education

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A HERMENEUTIC APPROACH TO HISTORY AND EPISTEMOLOGY IN MATHEMATICS EDUCATION: THE CASE OF PROBABILITY

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The paper presents a theoretical framework for a hermeneutic approach based on the interpretative line proposed by Bagni (2009). Our purpose is to show how this approach may serve as the basis of work with historical sources in teachers education. Its advantage is that it enables to present also teachers positions incommensurate with current mathematical discourse, thus relaxing the need for coherence imposed by an epistemological approach to learning. As an example of the approach, we look at the history of probability.

In his work *Interpretazione e didattica della matematica: una prospettiva ermeneutica* (Bagni, 2009), the Author suggests a shift from epistemology to hermeneutics in mathematics education. Indeed, contrasting Jahnke's hermeneutic approach (Jahnke et al., 2000) - which is Gadamer inspired and needs to see hermeneutics having an episodic character in the epistemological "stream" - Bagni's approach is Rorty inspired and allows to see hermeneutics in antithesis to epistemology. This approach authorizes to propose a radical pragmatic-hermeneutical approach to mathematics education. Although we use here historical sources to explain the research of interpretation tools, the subject works to each kind of text.

As investigative tool Bagni proposes an adaptation of Peirce's semiotics. At the basis of this semiotic approach we find the semiotic triangle but, from a global point of view Peirce's semiosis is a potentially unlimited process leading to the progressive construction of the meaning of a dynamic object. Bagni shows how the initial sign, which allows to start the semiotic chain, is comparable to an initial attitude (habit), and what Peirce calls "the final logical interpretant", can be seen as a mental "effect" (habit change) (Bagni, 2009, p. 212). We can explain this in the following matter. Facing the historical source, the subject is obligate to investigate the beliefs that induced the Author to formulate the sentences. She/he will do this according to her/his current beliefs and this may produce an awareness of the absence of an adequate knowledge, necessary for the interpretation; this allows to start the semiotic chain of meaning construction. The "habit change" would be a new, more meaningful attitude to face the text. The semiotic chain can be repeated using other sources until the subject judges the new attitude adequate to face a didactical processing of problems which treats the matter. This explains why the approach is able to produce and interpret changes in teachers' beliefs (Goldsmith et al., 2014) about mathematics, even when those changes involve radical reorganization of their system of reference: the reorganization is interpreted and measured by the acquired ability and don't' refer to real, objective values.

Probability is a meaningful example in this sense; its history has experienced at least two major epistemological ruptures, which teachers don't always seem to be aware

of. The first, with Buffon (1777), involves a shift in focus from discrete situations common in classical treatments to continuous ones with a concomitant shift in operational tools, namely, from arithmetic to geometric tools. The second, with Kolmogorov (1933) and his axiomatization, which completely changes point of view and leaves so aside the question of the nature of probability. Furthermore, the asking of an answer of the last question, which can be accomplished starting from irreconcilable philosophical and epistemological assumptions (Cera, 1990), was an obstacle for the construction of mathematical theory of probability and can be seen as an epistemological obstacle (Bachelard, 1938). We suppose also that the history of probability provides a good example of epistemological ruptures arising from a cultural substrate and we are convinced that teachers' difficulties with probability, beyond those arising from an inadequate mathematical background (Stohl, 2005), may be ascribed to obstacles in the interpretation of probability concepts. Our treatment may thus contribute to the debate concerning the theory of epistemological obstacles as it appears in the work of Guy Brouseau (Perrin-Glorian, 1994) and in Luis Radford's Cultural Semiotics (D'Amore, Radford & Bagni, 2006).

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